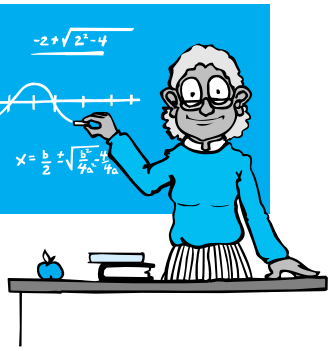

Now Here Is That Authority on Mathematics Reform, Dr. Constructivist!

Michael G. Mikusa and Hester Lewellen

Dr. Constructivist is a character who was "born" while we were trying to develop ways of teaching what constructivism is and ways to address the obstacles that teachers might have to overcome in adapting a mathematics-reform and constructivist approach in the classroom. We were both working with in-service and preservice teachers at all levels and found that the ideas of constructivism were still new to them.

We acknowledge that many definitions of the term *constructivism* exist. We believe, however, that a certain core belief is common to all definitions: that students actively construct their own knowledge of mathematics. Students are not blank slates on which a teacher can write. They cannot make a mental copy of the teacher's knowledge of mathematics. They cannot passively absorb skills and concepts in the form transmitted by the teacher. Instead, they have to struggle to make sense of them in a personal way. This idea, a product of recent research in cognition, seems to be the most difficult one for teachers to grasp, perhaps because they expect that constructivism will determine an alternative pedagogical method, which it does not do. Nevertheless, although constructivism says that mathematical knowledge cannot be transmitted directly from teacher to student, the teacher is not helpless to aid students in acquiring their knowledge.

Many definitions of the term constructivism exist

Because we are convinced that constructivism now offers the best explanation of how students learn, we were tempted to "sell" this way of looking at student learning, but we know that selling entails the risk of converting some people while alienating others. We decided to find a more playful way to introduce these ideas.

The character that we created, Dr. Constructivist (Dr. C), is supposedly an aged authority on mathematics education, one who has witnessed nearly a century of evolution in the theories of human learning. We see her as an intermediary who responds to letters from teachers that represent various stages of acceptance of the ideas of constructivism. This role gives her a chance both to explain constructivism to the uninitiated and to offer helpful suggestions to those who run into difficulties while attempting to institute changes in their own classrooms.

The letters that we created describe real situations, but we wrote them with a light touch. When we brought Dr. C to life at conference presentations, the letters gave listeners a chance to identify with the writer and laugh at the same time. Dr. C's answers allowed us to explore alternative modes to traditional teaching.

It now seems appropriate to share these letters with a wider audience. The aim is to open a forum for a national discussion of mathematics reform and constructivism. To this end we have created a home page for Dr. C on the World Wide Web. The address is as follows:

monster.educ.Kent.edu/~mmikusa
/Dr.Constructivist.html

The Dr. C letters and responses that follow are not all-encompassing, nor are they the last word. They are intended merely to stimulate discussion. We can envision their use in a variety of situations, with or without the accompanying responses from Dr. C.

THE DR. CONSTRUCTIVIST LETTERS

Dear Dr. C,

I have been teaching ninth-grade mathematics for the last four years and have been trying to get my students to think mathematically. But when I ask them to explain their answers or ask how they found

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an answer, they just repeat the procedure that they followed. Although I think that this response is adequate, I would like to know how I could get them to reflect on their thinking or, even better, think about others' thinking. Please help me on this matter, Dr. C!

Desperately,
INEEDA ANSWERS

Dear Ineeda,

The problem is that when your students respond at the level of facts and procedures, you believe that this response is adequate. You need to be dissatisfied with these responses. For example, if your students repeat the steps of a procedure instead of giving an explanation, ask them how they know that the procedure is correct. Do not accept external authority—for example, "you told us . . ." or "the book says . . ."—as a legitimate source of justification. Require pictures, demonstrations with concrete models, or some other type of informal or formal mathematical reasoning rather than these referrals to authority.

Dear Dr. C,

I know that my students should do more discussing, conjecturing, and justifying during class, but I find it difficult to start these processes in my classroom. When discussion does occur in the classroom, it usually involves the same six to eight students. How can I create the kinds of questions that will encourage more students to participate in this learning process?

Perplexedly,
JUST BEGINNING

Dear J. B.,

Many good ways exist to involve your students in these discussions. Choose questions that are more open-ended. Open-ended problems are usually not subject to a "right" or "wrong" label. Be sure to prompt all your students to respond. Emphasize the value of sharing students' thinking with the class no matter how "off the wall" a response may seem. Your goal during the beginning of the year may be to have students see the importance of considering other interpretations of mathematics problems and mathematical ideas. In other words, change students' definitions of mathematics to emphasize exploration of problems rather than attainment of answers.

Dear Dr. C,

I have been using a discovery approach in teaching for about two years, and I have noticed a problem that I would like you to help me solve. When doing free explorations, my students sometimes make discoveries other than the ones that I intend for them to make, so I have to guide them to make the intended discovery. Furthermore, when I assess my students on these ideas, I find that they do not score well on the tests. What can I do?

Earnestly,
WILL NOT DISCOVER

Dear Will,

When you assess your students, are you assessing them on the discoveries that you wanted them to make or on the discoveries that they did make? If they really did make mathematical discoveries, why not assess them on their discoveries? Isn't the point of assessment to learn what mathematics they do know?

Perhaps the activity that you used for this lesson caused students to be more divergent in their thinking. If you wanted your students to consider specific mathematical ideas and they found it easier to consider others, have them answer specific questions regarding the content that you wish them to explore. For example, ask them to record particular data and answer specific questions about the content before they do their free exploration.

Our character, Doctor Constructivist, responds to teachers in different stages of accepting constructivism

Dear Dr. C,

I have been wondering what all the talk is regarding constructivism. I have taught mathematics in secondary school for about ten years and have had great success. I find that my students like me and how I teach. Furthermore, my students score well on standardized tests, are accepted by many fine colleges, and do very well in college mathematics. How could this be, as I have never read the NCTM's Standards and I do not know what they say about how students learn?

Scornfully,
TERRY TRADITIONAL

Dear Terry,

You say that you are wondering about constructivism. Constructivism is not a new prescription for teaching mathematics. It is simply a theory about knowledge and learning—with implications for teaching. Constructivism has grown out of the synthesis of cognitive psychology, philosophy, and anthropology. The theory defines knowledge as temporary, developmental, and socially and culturally mediated. Learning from this perspective is understood as a self-regulated process of resolving inner cognitive conflicts that often become apparent through concrete experience, collaborative discourse, and reflection.[1]

[1]This description of constructivism is from Catherine Twomey Fosnot's preface to *The Case for Constructivist Classrooms*, by Jacqueline Grennon Brooks and Martin G. Brooks (Alexandria, Va.: Association for Supervision and Curriculum Development, 1993).

I have no doubt that all that you say about your classroom is true. Your students like you—Good!—they pass standardized tests—Fine!—and they are accepted into great colleges. Have you considered that you might actually be implementing many of the suggestions of the NCTM's Standards even though you have never read them? Surely we cannot say that no good teachers existed before the NCTM's Standards were created.

We would like you to reflect on this question: Regardless of what the standardized tests say about your students, do you really believe that your students understand mathematical concepts and can use them to solve real problems that arise in their lives? If your answer is yes, you will find that the NCTM's Standards support your teaching style. If your honest answer is no, we suggest you read the NCTM's *Standards* documents (NCTM 1989, 1991, 1995).

Base your assessment of learning on the mathematics that your students do and say

Dear Dr. C,

I began teaching because I wanted to help students learn mathematics. I thought that helping my students learn mathematics meant that I would show them such techniques as my eight-step plan for solving quadratic equations or demonstrate such skills as factoring the difference of two cubes. I believed that I was helping my students because I could see them performing these tasks. However, I have been told that my teaching is not consistent with constructivist beliefs. How can I help my students construct mathematical ideas, and how will I know whether they have learned mathematics?

Sadly,
SAY IT AIN'T SUE

Dear Sue,

We feel your pain! It is clear that you love your students and want them to learn skills and procedures that are valuable techniques for solving problems in mathematics. However, we believe that your students should learn these skills and procedures in a context where they see the relevance and necessity of learning them. Constructivist theory suggests that when students are given interesting and relevant problems, problems that they do not have a direct method for solving, they will either find alternative methods for solving these problems or create a personal context in which to learn the procedures and skills most applicable to the situation.

You need to present students with mathematical problems to investigate *first*, so that students can develop a *need* for the procedures and skills that are appropriate for them to learn. Students might not learn every skill and procedure, but so what? Students who have memorized procedures without a personal context often have difficulty remembering and using this mathematics, so your students will still come out ahead. Of course, it is not easy to say good-bye to your eight-step method for solving quadratic equations, good-bye to interpolation, and good-bye to taking square roots by hand. But their only purpose now is to demonstrate historical context or to prove to your students that mathematics existed before calculators.

Now to your second question, about knowing how to tell when your students have learned mathematics. I would say that you should base your assessment of mathematics learning on the mathematics that your students do and say. Listen to your students' questions and to their responses to your questions and those from other students. If your students' questions are at the level of simple recall and their responses are lists of facts or a repetition of steps in a procedure, you have not helped them acquire mathematical power. However, asking what would happen to a problem that they just solved if some detail was changed is a good sign. If, on their own, your students make a new connection between two mathematical ideas, like the distance formula and the Pythagorean theorem, that too is a good sign. If you find yourself saying, "I never thought of solving the problem like that" or being amazed by your students' creativity, those are good signs. Well, I could keep adding to this list, but I think that you get the idea.

Dear Dr. C,

When I was a student, I hated group work because I did all the work and the other people in the group just copied my answers and got full credit. Now our new principal wants us all to use cooperative groups because he says that method is more democratic. I think that I would just be punishing my better students. What do you say?

Proudly,
800 ON THE SATS

Dear 800,

If cooperative groups mean that one person in the group does all the work, we do not like it either. Clearly, you believe that your administrator is coercing you to engage in a teaching style with which you are uncomfortable. You feel that you are at odds with the concept of cooperative groups both personally and pedagogically.

However, more than one approach to cooperative groups exists, and you may be able to find alternatives that will satisfy you, as well as your principal. Read articles that explain alternative ways of having students work together, and try some of the suggested ideas. For example, some teachers like to assign particular roles to each student in the group, roles that rotate regularly. You can also experiment with different group sizes. As students become con-

fident of their ideas within their groups, you can encourage discussions among groups.

I would like to make two key points, however. The first point pertains to the choice of the collaborative activity itself. It should be an activity that takes more than one mind to figure out, not something that you could assign for individual work. It should be relevant to students and thereby motivating in itself. It should involve important mathematical ideas, not be merely a game with scant relationship to the mathematics to be learned. It should move the curriculum forward, not put it on hold. You need to find or create interesting and challenging activities that encourage students to think, discuss their thinking, and justify their mathematical ideas.

The second point pertains to the need for debriefing. Students should have an opportunity to discuss their results as a class. If their results conflict, they should have an opportunity to convince one another that their own thinking is correct. This process of creating convincing arguments or being convinced by others helps students construct mathematical ideas.

Dear Dr. C,

In a secondary methods class, we watched a videotape of some second graders learning two-digit subtraction in which the teacher never said whether a student's answer was right or wrong. I am wondering how to apply these techniques to teaching high school algebra, for instance, factoring trinomials. All those elementary students seemed to get the right answers eventually. What should I do if my students agree on a wrong answer? Should I tell them that their answer is incorrect? Will I be doing serious damage to their mathematical power?

Nervously,
IDA KNOW

Dear Ida,

You were obviously watching one of the videotapes made by Constance Kamii to demonstrate constructivist teaching. For those of you who have not seen this particular tape, the scene is a second-grade classroom in which the teacher presents the class with two-digit arithmetic problems that they have never seen before and then asks them the best way to get the answer to the problem by thinking about it. The students, who do not have pencil and paper, suggest many different answers. By the time that the students have heard a number of explanations, they have agreed on the correct answer. You are wondering what to do if your high school class agrees on an incorrect answer.

These situations will certainly happen from time to time. Try to present another problem that is related to the agreed-on problem and that will cause students to think about the original problem. For example, suppose that your students agree that the way to solve the problem $x^2 - x = 6$ is the following:

$$x^2 - x = 6,$$
$$x(x - 1) = 6,$$
$$x = 6$$

or

$$x - 1 = 6.$$

Therefore, $x = 6$ or $x = 7$.

When students check the problem, they discover that these answers do not satisfy the original equation. You may then offer a similar problem, such as $x^2 - x = 0$. If your students employ the same technique, they will find that these solutions work. You can then ask them to tell you how the two problems are alike and how they differ. This common mistake, where students do not understand the uniqueness of zero in the zero-product property, is familiar to all teachers of algebra. After your students compare the two equations and see that the only difference is that one equation is set equal to 0 and the other to 6, you may wish to present numeric examples that help them make sense of this difference. For example, ask them to find as many pairs of numbers that satisfy the equation $a \cdot b = 6$ and another set such that $a \cdot b = 0$.

$a \cdot b = 6$		$a \cdot b = 0$	
a	b	a	b
1	6	1	0
2	3	0	–6
–3	–2	9	0
–6	–1	0	0

After students have shared their factor pairs with the class, ask them to explain why they set each factor, x and $(x - 1)$, equal to 6 in solving $x^2 - x = 6$ or equal to zero in $x^2 - x = 0$. Finding new questions to provoke student thinking is preferable to telling students what to do.

The result of telling your students which answers are right and which are wrong is that you will become their mathematical power and they will depend on you to think for them. Your best option is to present them with questions that allow them to make sense of the mathematics that they are learning for themselves.

The activity should move the curriculum forward, not put it on hold

Dear Dr. C,

I have been teaching for fifteen years and have recently been trying to encourage my students to be more active learners. I find that I am giving fewer "how to do it" lectures and requiring my students to think more. I am having a problem with complaints from students and parents. They say that I am not teaching because I do not give my students step-by-

step procedures showing how to solve the problems. Moreover, sometimes I think that they are right! What shall I do?

Fretfully,
ABUSED N. CONFUSED

Dear Confused,

Oh, these workaholic mathematics teachers! They believe that if they are not doing all the work, they are not earning their salary! Naturally the students agree!

Please remember Dr. C's maxim: Reinventing the wheel is stupid if you are interested only in wheels. As teachers, however, we are also interested in invention, especially if we believe that all students must invent their own understanding of mathematical processes and concepts.

This maxim does not discount the importance of skills and procedures; they are the wheels on the car that takes us where we want to go. But the critical thinking that goes into making personal sense of the mathematics is the invention that gives students confidence in their own abilities to do and use mathematics—the ability to drive the car, not just be a passenger.

The real problem here is beliefs. Students, parents, and—as you confess—even the teacher have strong beliefs about how a mathematics class should proceed. The common belief is that the teacher is the authority who can dispense knowledge; the students are there to memorize and spit back the correct skills and procedures. These common beliefs are being turned upside down by mathematics reform, which desires that students become mathematical authorities and that memorization should take a backseat to understanding. I am sure that you can figure out for yourself all the various psychological reasons that teachers, parents, and students may oppose this change—shall I say, why they need convincing that change would be an improvement.

Skills and procedures are the wheels on a car that takes us where we want to go

But you must be the first one convinced. The best way to convince yourself about what you should be doing is to read the research, apply some suggestions from the research in your teaching, and reflect on changes in your students and your teaching. Let this personal-action research guide your beliefs about teaching and learning.

Dear Dr. C,

It is already May. I have been having so much fun with my classes, playing with manipulatives, using technology, and creating exciting activities to present mathematics, that I will not be able to finish the whole curriculum in my course. My principal is furious. What do I do now?

Playfully,
BEHIND THE EIGHT-BALL IN BUCYRUS

Dear Behind,

One needs to keep a sense of balance in one's teaching. The new methods of teaching mathematics are fun, but they should be stepping-stones to accomplishing your curricular goals. You may need to start thinking now about next year and do a better job of planning. As far as this year's students are concerned, in the time that remains, try to pick a few topics that are broad enough to include multiple objectives, for example, combining geometry, probability, and problem solving in a unit plan. Be creative! Remember that creating mathematically powerful students is a major goal for any mathematics curriculum.

Dear Dr. C,

I have been troubled about the proposed Standards since I first read the introduction to the Curriculum and Evaluation Standards for School Mathematics *(NCTM 1989). The part about students becoming autonomous thinkers has me worried. Are you sure that businesses and politicians want people who can think for themselves? Would thinking people not disrupt the status quo?*

Impudently,
TONGUE N. CHEEK

Dear Cheek,

Regardless of what you think about students becoming autonomous thinkers, we believe that helping our students become mathematically powerful is our most important goal—even if it means shaking up the establishment!

Actually, you have raised an issue that is appearing in newspapers all over the country. Letters to the editors of various newspapers have implied that mathematics is only arithmetic, and furthermore, some people seem to think that the aim of education is to stamp out educational products the way that a factory stamps out cars. Well, if you have visited factories and businesses recently, then you know that they require much more than repetition of mindless tasks. Most businesses now require teamwork, problem solving, and the ability to deal with and communicate quantitative information. All such businesses require and have available computers and calculators to do the computations, but they need employees—our students—to know how to use such devices, when to use such devices, and whether the information that they are getting from these tools is meaningful.

These changes are inevitable. Perhaps it is time to look at where we want our country to be in the next ten or twenty years and face up to changes in mathematics curriculum and instruction that will be necessary to attain these goals.

We think that the Dr. Constructivist letters speak for themselves, but we reiterate the idea that many real obstacles impede mathematics reform, not the least of which are teachers' beliefs about what mathematics is, how students learn, and how one knows what mathematics has been learned. Nevertheless, we believe that constructivism is the best theory of learning available now. It is supported by ongoing definitive research in cognitive psychology and mathematics education. As caring professionals, we believe that we are obligated to incorporate this knowledge into our work. The practice of using knowledge from research to refine our teaching, and then evaluating its effects on our students' learning, will not only help us improve as individual teachers but simultaneously expand and refine our knowledge of the teaching and learning of mathematics in general.

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DISCUSSION QUESTIONS FOR "NOW HERE IS THAT AUTHORITY ON MATHEMATICS REFORM, DR. CONSTRUCTIVIST!"

1. After reading Dr. C's answer to the question from Ineeda Answers, can you give a recent example from one of your classes of a shallow "the book says so" type of answer? What had you hoped to hear? How can you change your questions to obtain a better response from students?

2. After reading the question from Will Not Discover, decide whether every lesson should be based on discovery. Can you give examples of lessons and topics that should not be discovery topics?

3. After reading the question from Terry Traditional, can you point out some methods that traditional teachers already use that help students construct understanding?

4. After reading the question from 800 on the SATs, decide why a discussion helps people construct their own understanding of ideas. How often should students work in groups if group work is to be effective?

5. Some people refer to the constructivist approach to teaching as "fuzzy math" because students need to go beyond simple correct answers. Discuss how constructivist teaching might be misinterpreted as "fuzzy." How can teachers misuse constructivist ideas to make their teaching really fuzzy? In what ways can you take advantage of constructivist theory to avoid being fuzzy?

6. What is the difference between group work and constructivism?

If you are like Just Beginning, who is about to start and manage a class discussion, look at *Every Minute Counts: Making Your Math Class Work,* by David Johnson (Palo Alto, Calif.: David Seymour Publications, 1992). The chapter called "The Art of Questioning" has some great tips about classroom discourse.